A Spatial Price Equilibrium Based Navigation System NED Model For the UMR-IW Navigation System Feasibility Study

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1. Introduction

The purpose of this paper is to present the National Economic Development [NED] model used in the evaluation of potential changes to the locks of the Upper Mississippi River - Illinois Waterway (UMR-IW) Inland Navigation System. The NED evaluation is conducted as an integral component of The Upper Mississippi River - Illinois Waterway Navigation System Feasibility Study as required by Engineer Regulation (ER) 1105-2-100, 28 December 1990. This planning guidance ER implements for the Corps of Engineers the Economic and Environmental Principles for Water and Related Land Resources Implementation Studies, February 3, 1983, and the Economic and Environmental Guidelines for Water and Related Land Resources Implementation Studies March 10, 1983 published by the Water Resource Council. The principles define the Federal objective of water and related land resources project planning as to contribute to national economic development consistent with protecting the Nation's environment. The guidelines describe how Federal water resource planning is to be conducted, detail procedures and a system of accounts for display of the economic, social, and environmental evaluations, and outline a plan formulation process.

Contributions to the NED account of a Federal water resource action, project, or plan are defined as increases in the *net* value of the national output of goods and services. The use of the net increase in the value of goods and services recognizes that water resource projects have both beneficial and adverse effects in the national economy. Beneficial effects in the NED account are increases in the economic values of national output of goods and services from implementing a plan, the values of outputs resulting from external economies caused by a plan, and the values associated with the use of otherwise unemployed or under-employed labor resources. Adverse effects in the NED account are the opportunity costs of the resources used in implementing a plan. These adverse effects include the direct outlays required for implementing the plan, associated costs created by the implementation of the plan, and other direct costs required by the implementation of the plan.

The procedures for evaluating the NED effects of plans are detailed in Chapter 6 of ER 1105-2-100. The ER recommends a comparative static technique to identify the changes in the NED account resulting from a Federal water resource action or plan. First, the without project condition is identified and the NED account is evaluated. Then, the with-project condition is identified and the NED account is evaluated with the project, action, or plan in place. Finally, the difference between the NED account with the project in place relative to the without project condition is identified as the net contribution of the project to the NED account. As an additional complication, most water resource projects have useful lives extending many years into the future. Consequently, the beneficial and adverse effects on NED aren't necessarily coincident through time and the without and with-project conditions must be forecast at selected points in the future. The net NED impacts are estimated at the selected points in time and then the resulting values are discounted to a common base year to measure the net present value of NED effects. These net present values are then typically displayed as average annual values.

The general measurement standard for the values of goods and services created by a Federal water resource action, project, or plan is defined by ER 1105-2-100 as the willingness of users to pay for each increment of output provided by a plan. Since it is not normally possible to directly observe or measure the full value of incremental output to users, four alternative measures for estimating the willingness of users to pay for incremental output are described in ER 1105-2-100. The four alternative estimates of the willingness of users to pay for incremental outputs are (1) market prices paid by users, (2) changes in users' net income, (3) costs of the most likely alternative to use, and (4) administratively established values. The ER also provides that innovative procedures designed to more accurately estimate the NED effects of plans may be employed if the new procedures are fully documented.

The relative efficacy of the alternative measures of willingness to pay depends on the quantity and type of incremental output provided by a plan. For example, if the additional output from a plan is too small to have an effect on observable market price, then market price closely approximates the willingness to pay for incremental units of output. If the increased output of the plan will have a significant impact on market prices, then estimated prices for each increment of output are needed to derive the total value of the incremental output. If the output of a plan is intermediate goods or services used by producers in the production of other goods and services, then the change in net income of the producers created by the incremental intermediate outputs of the plan is an appropriate measure of willingness to pay. If the outputs of a plan replace some other good or service, then the difference in the costs of the replaced output relative to the plan costs is a useful measure of the willingness to pay. Finally, in situations where plan outputs aren't marketed goods, then administratively established values may serve as proxies for social values of incremental output.

Section VI of Chapter 6 of ER 1105-2-100 describes the specific procedures for measuring the beneficial contributions to NED associated with the inland navigation features of water resource projects and plans. The fundamental economic benefit of a navigation project is defined as the reduction in the value of resources required to transport commodities. Four categories of navigation benefits are established and defined dependent on the without project condition for potential project beneficiaries. These categories are: (1) cost reduction benefits; (2) shift-of-mode benefits; (3) shift-of-origin-destination benefits; and (4) new-movements benefits.

The cost reduction benefit category measures the reduction in transportation costs for traffic that would use the waterway in both the without and with project conditions. The reduction in resources required to accomplish the movement in the with project condition relative to the without project condition represents a NED benefit because resources that would be needed to accomplish the movement without the project are released for productive use elsewhere with the project in place.

The shift-of-mode benefit category measures the reduction in transportation costs required to accomplish a movement with the project in place relative to the without project condition, when the movement would use an alternative mode of transportation from the same origin to the same destination in the without project condition. Here, the NED gain is the reduction in the cost of resources required to accomplish the movement on the waterway with the project in place, relative to the more costly movement on the alternative mode without the project.

The shift-of-origin-destination benefit category measures the increase in the NED account created by a commodity movement originating or terminating at a different location with the project in place relative to the without project condition. For a shift in the origin of a commodity movement, the beneficial NED effect is the difference in total resource costs of moving the commodity to its place of ultimate use with the project relative to without the project. For a change in the ultimate destination of a commodity movement, the beneficial NED effect is the increase in net revenue to the producer of the commodity arising from the with project destination relative to the without project destination.

The new-movement benefit category measures the increase in the NED account attributable to commodity movements that only occur with the project in place. This category measures the value of the increases in production that would not have occurred in the without project condition and only become economically profitable with the decreased transportation costs arising from the project or plan.

No matter which measure or category of the NED benefits is appropriate, the method for computing the NED benefits in inland navigation studies is given in ER 1105-2-100 as an iterative procedure with ten distinct steps. The level of effort expended on each step depends on the nature of the proposed navigation system improvements, the state of the art for accurately measuring the NED estimates, and the sensitivity of project formulation to further refinements in data or analysis. The ten steps are:

Step 1 Identify the Commodity Flows;
Step 2 Identify the Study Area;
Step 3 Determine Current Commodity Flows;
Step 4 Determine Current Costs of Waterway Use;
Step 5 Determine Current Cost of Alternative Movement;
Step 6 Forecast Potential Waterway Traffic by Commodity;
Step 7 Determine Future Cost of Alternative Modes;
Step 8 Determine Future Cost of Waterway Use;
Step 9 Determine Waterway Use, With and Without Project; and
Step 10 Compute the NED benefits.

Steps 9 and 10 are the focus of this document. Steps 1 through 8 are addressed in other study documents.

2. Economic Theory

The fundamental economic benefits afforded by the inland navigation transportation system in general and the UMR-IW system in particular are the movement of goods from geographic areas where they have relatively low economic value to areas where they have relatively high economic value. The relationship between the price of inland waterway transportation service and the amount of service desired by potential users defines a demand function for inland waterway transportation services. The demand for inland navigation transportation services is termed a derived demand. It is derived from the difference in value of a good amongst spatially separated locations. For example, the fundamental reason that there is a demand for transporting farm products down the Mississippi River from the Midwest to the Gulf Coast is that farm products have a greater value at the Gulf Coast than they do in the Midwest. Consequently, potential shippers of products are willing to pay for the provision of inland waterway transportation service. Furthermore, the products transported on the inland navigation system are, typically, intermediate products destined for use as an input to production of other final consumer goods. These ultimate consumer goods must then be transported to consumers. Hence, consumers do not generally use inland waterway transportation as a final end product that directly affects their welfare. These facts suggest an economic perspective of the inland waterway transportation system as a component of a larger transportation network linking producers to other producers and, ultimately, producers to final consumers.

The fundamental economic costs of the inland navigation transportation system are the opportunity costs of the resources consumed in producing the inland waterway transportation service. The relationship between the resources required to produce various levels of inland waterway transportation services defines a production possibility frontier for waterway transportation services. Some of the resources required to produce inland waterway transportation are supplied by the public sector and other resources needed to produce inland waterway transportation are supplied by the private sector. Federal, state and local governments provide much of the infrastructure necessary for navigation through the inland waterway system. Examples of publicly provided resources include dams, locks, regulating works to maintain channel depths, public docks, and other infrastructure necessary to operate and maintain the navigation system. The private sector provides the floating plant, other equipment, private docks, fuel, labor and other inputs necessary to accomplish the actual movement of goods. Consequently, inland waterway transportation is a jointly produced output of the public sector and the private sector.

It is important to note that given levels of output can be produced using very different combinations of inputs. In a very real sense inputs are substitutes for each other. For example, to produce a given level of output it is possible to use different combinations of infrastructure, fuel, labor, and tows. Since it is possible to substitute inputs in the production of waterborne transportation, the efficient (low cost) combination of inputs needed to produce any level of output depends on the prices of the inputs as well as the technical combinations that can be used to produce any level of output.

The relationship between the production possibility frontier, the cost of resources, and the industrial organization of private sector providers defines a supply function of inland waterway transportation services. The supply function of inland waterway transportation services reflects the relationship between the transformation of resources (economic inputs) into inland waterway transportation services and the price of those transportation services. The supply function formalizes the relationship between the quantity supplied of a good or service and the price of the good or service. Private sector economic organizations supply inland waterway transportation services in response to the demands for the services. Private sector providers of inland water transportation do so in the quest for economic returns, profits, for their efforts. The private sector costs of providing inland waterway transportation depend on the performance of the publicly provided components of the system. For example, the better the performance of inland waterway system locks, the more quickly tows may move through the system and, consequently, the more output that can be provided by each unit of the private sector owned equipment. Hence, the privately supplied quantity of inland waterway transportation depends on the performance of the publicly supplied components of the inland waterway navigation system.

There is a vast body of literature describing the economic impacts of freight transportation systems in the larger economy. A bibliography of much of this literature is provided as an attachment to this document. Samuelson (1952) introduces the analytical framework of modeling transportation's role in the larger economy as critical for evaluating the contributions of transportation to the economy at large. Harker (1985) provides a very good summary of alternative modeling approaches and the advantages of each. Waquil and Cox (1995) present a spatial price equilibrium model with intermediate and final goods designed to evaluate policy regarding some South American agricultural markets

The consensus of this literature is that the economic impacts of transportation systems are best analyzed as components of larger spatial price economic models. Analyzing transportation systems or their individual components myopically can lead to erroneous conclusions regarding economic impacts and values of the transportation system and its components.

Spatial price economic models may be characterized as models where consumers' demands for and producers' supplies of goods and services are identified by their location in spatially separated geographic regions called markets. The spatially separated markets are connected by transportation links. The links connecting markets are differentiated by the markets they connect and the mode of transportation they afford. The spatially separated supplies and demands for goods and services induce a derived demand for transportation services by shippers on the links of the transportation network. The induced demand for transportation services by shippers on the various links comprising the transportation network is satisfied by the supply of transportation services produced by carriers on the links. The supplies of transportation services by carriers on the links of the transportation system are influenced by the performance characteristics of the links. Prices serve as signals to producers, shippers, and carriers conveying information and coordinating their decentralized decisions regarding levels of outputs.

An important class of spatial price economic models is generalized spatial price equilibrium models. Generalized spatial price equilibrium models seek to identify the balance between supply and demand in spatially separated product markets as well as the balance between supply and demand on the transportation links connecting the spatially separated product markets. The balance between the opposing forces of supply and demand in an individual market is termed market equilibrium. A market equilibrium is characterized by an equilibrium market quantity and an equilibrium market price. The equilibrium market quantity is the market quantity where the quantity demanded is equal to the quantity supplied. The equilibrium market price is the price that brings about the equalization of the quantity demanded with the quantity supplied. Spatial price equilibrium models seek to find the simultaneous equilibrium of spatially separated product markets in conjunction with an equilibrium in markets for transportation services on the links connecting the product markets.

Figure 1

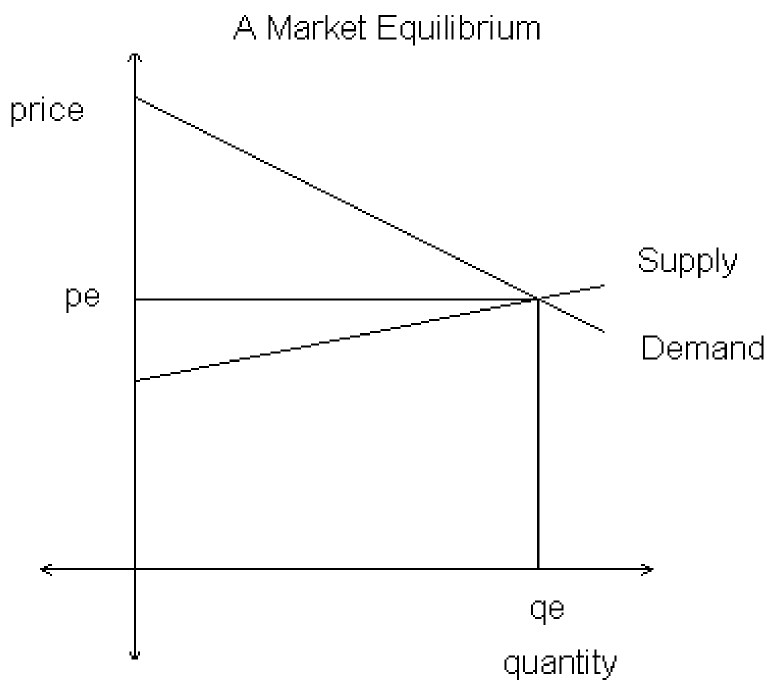


Figure 1 presents an equilibrium condition in a typical market. Graphically, the equilibrium quantity and price are determined as the point of intersection of the market supply and demand curves. In Figure 1, the equilibrium quantity is qe and the equilibrium price is pe. Only at price pe is the quantity demanded equal to the quantity supplied. This price balances the opposing forces of supply and demand in this market. A simultaneous equilibrium is a set of prices and quantities that balances supplies and demands across related markets.

Spatial price equilibrium models are useful in the comparative static framework suggested by the planning guidance to evaluate the NED changes resulting from potential changes in the performance of components of the inland navigation system. This is especially true when the navigation system under evaluation is geographically expansive, the navigation system is heavily used, and the navigation system transports intermediate products to producers destined for final consumption at locations far removed from the navigation system. The UMR-IW navigation system meets these criteria.

The NED evaluation of any potential plan, action, or project at selected points in time is accomplished by first solving for the spatial price equilibrium without the potential UMR-IW system action in place. Then the potential system action is incorporated in the model altering the performance and productivity of the UMR-IW navigation system and, consequently, the supplies of transportation services offered by carriers on the UMR-IW navigation system. A new spatial price equilibrium arising from the altered performance of the transportation system is then estimated and compared to the original equilibrium to evaluate the resulting changes in the NED account generated by the system action.

Figure 2

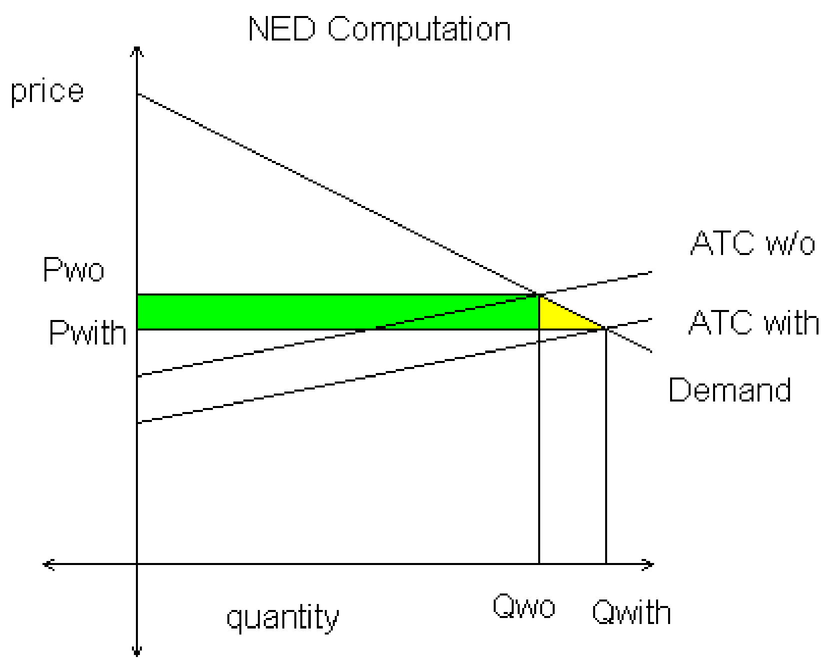
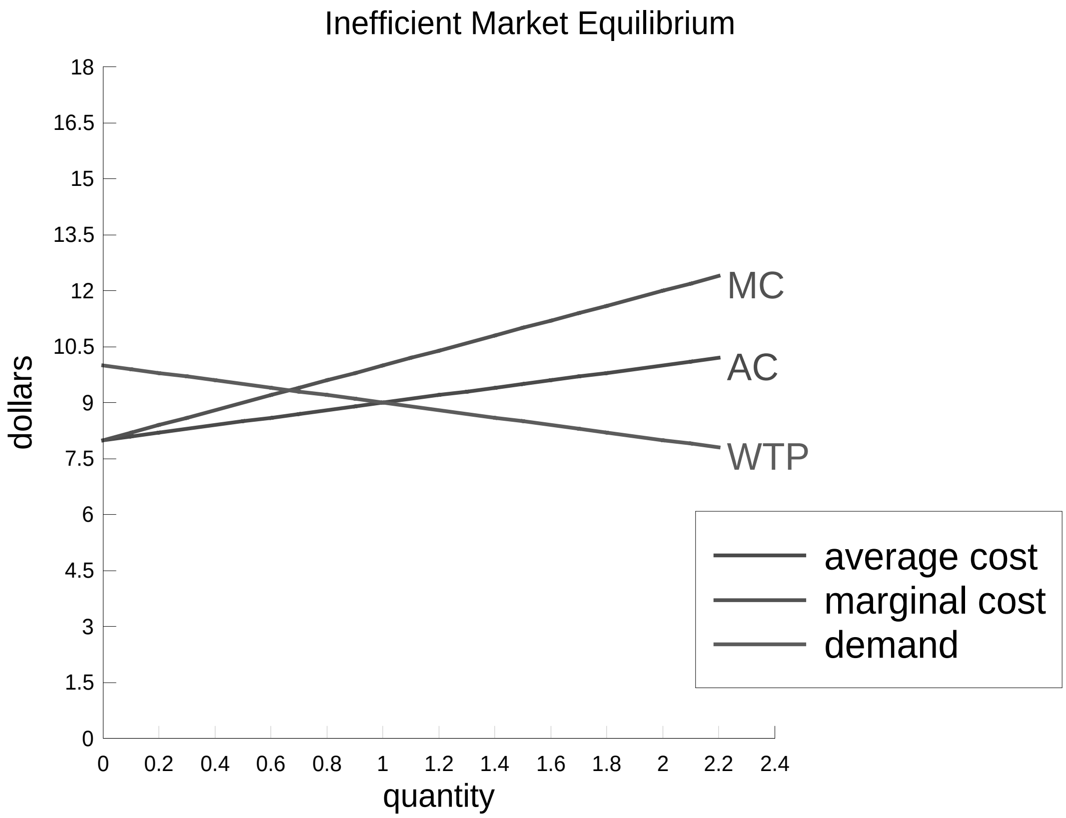


Figure 2 above depicts the computation of the NED benefits of an action that reduces the costs of transportation in a given market. In Figure 2 the curve labeled $ATC_{w/o}$ depicts the average cost of transportation without the action and ATC_{with} depicts the average cost of transportation with the action in effect. Both ATC curves are drawn with positive slopes to indicate that average transportation costs increase in this market with the quantity supplied. The ATC_{with} is located below the $ATC_{w/o}$ to indicate that for any quantity of transportation supplied in the market, the action reduces the average cost of transportation. The net NED benefits generated by this market without the action in place are measured by the area under the demand curve from the origin to the quantity q_{wo} net of the area of the rectangle formed by the product of p_{wo} and q_{wo}. The area under the demand curve represents the total willingness to pay for quantity q_{wo} and the area of the rectangle represents the opportunity cost of the resources used in supplying the quantity q_{wo}. The net NED benefits generated by this market with the action in place are measured by the area under the demand curve from the origin to the quantity q_{with} net of the area of the rectangle formed by the product of p_{with} and q_{with}. The difference in the net NED benefits between the without and with action conditions, represented by the shaded area in Figure 2, are the increase in net benefits attributable to the action. The portion of the shaded area composed of the rectangle $(p_{with} - p_{wo}) * q_{wo}$ represents the transportation cost reduction savings to traffic that will move in the without action condition. The remaining portion of the shaded area represents the total shift-of-origin-destination and new-movement benefit categories. Note that there are no shift-of-mode category benefits in the computation of the net NED benefits of this potential action. This is as expected because shift-of-mode category benefits are very unlikely to be the consequence of an action to improve a component of an existing transportation system. The effects of the improvement will occur at the margin and, consequently, shift-of-mode benefits are unlikely as transportation modal shifts rarely occur at the margin.

Note further that in Figure 2 the average cost curves are depicted as increasing functions of the quantity supplied. When this is the case, this market will lead to an inefficient equilibrium in the sense that total net NED benefits can be increased by decreasing the quantity supplied to a level where the *marginal* cost curve would intersect the demand curve. This situation is depicted in Figure 3 below. Whenever the average cost curve is increasing the marginal cost curve is located above the average cost curve. The quantity that maximizes the net NED benefits of this market is given where the marginal cost curve intersects the demand or willingness to pay curve. This is because quantities greater than the level where marginal costs equal the willingness to pay increase total costs more than total benefits. Hence, markets with supply curves defined by increasing average cost curves will not yield an efficient equilibrium without some intervention.

Increasing average cost curves in transportation markets are indicative of an industrial organization with many competitive suppliers whose individual use of the system creates external congestion costs for other system suppliers. This is likely the case for carriers operating on the inland navigation system.

Figure 3



3. The UMR-IW Navigation System NED Model

This section provides details of the definitions and postulates underlying the NED model created for the Upper Mississippi River - Illinois Waterway Navigation System Feasibility Study to measure the navigation related NED impacts of alternative plans. As the focus of the feasibility study is congestion at the locks in the UMR-IW navigation system the model is built around the performance of the system locks and the impact of the locks' performances on the supplies of inland waterway transportation by private system carriers. The carriers' supplies then interact with the derived demands of shippers to yield an estimate of the UMR-IW navigation system equilibrium commodity flows and prices. These estimated equilibrium market prices and commodity flows form the basis of the measure of the willingness of users to pay for incremental outputs, which, in turn, form the measure of net NED benefits created by the system. Hence, by changing the levels of demand to those levels forecast at selected times in the future and estimating the performance of system locks at those selected points in time, a sequence of system equilibria may be estimated. This sequence of equilibria yields estimates of the future NED benefits generated by the system. The change in net NED benefits created by implementing an action to improve the performance or reliability of locks at future points in time may then be estimated by comparing the without action system equilibrium and the with action system equilibrium at those points in time.

The system model may be characterized as a comparative static, simultaneous equilibrium model, where the system equilibrium is characterized by a vector of delays at system locks and a matrix of specific origin, destination, commodity flows. Lock delays occur when a tow arrives at a lock chamber that is already in use by another vessel. The lock delays reduce the quantity of output that system carriers can deliver per unit of time as equipment waiting for service at a system lock is not actively producing transportation service. The reduced output of equipment increases system transportation costs per unit of output for carriers, thereby decreasing the quantity of output supplied for all transportation prices. Commodity flows are then altered as system shippers adjust their demands in response to the prices charged by carriers. The system equilibrium is the level of system delays where the quantity supplied is equal to the quantity demanded for all river origins, destinations, and commodities.

The NED model developed for the UMR-IW Navigation System Feasibility Study embodies important economic results from spatial price equilibrium models. These results are employed as postulates enabling the construction of an economic model focused directly on the contributions of the UMR-IW navigation system to the NED account consistent with the integration of the transportation benefits into the national economy. The important spatial equilibrium results are described in detail below along with other significant definitions and assumptions employed in constructing the UMR-IW navigation system NED model.

Definition 1: *Producers* are private sector, economic agents that produce goods in spatially separated geographic regions. Producers can also be consumers of the intermediate goods transported on the UMR-IW. Producers' supply and demand decisions create the derived demand for transportation on the UMR-IW.

Definition 2: *Shippers* are private sector, economic agents that determine the quantities of goods to move between regions and the set of carriers, defined below, which will move the goods. Shippers are the economic agents that demand UMR-IW transportation services. Shippers perform an arbitrage service between regions. Shippers may also be producers as described in the definition of producers above.

Definition 3: *Carriers* are private sector, economic agents that provide transportation services on the UMR-IW navigation system and elsewhere. Carriers convert private sector inputs into private sector supplies of waterborne transportation. Commodity movements are accomplished by tows. Tows are comprised of barges and towboats in various configurations dependent on the commodity group transported by the tow. The supplies of tows by carriers are affected by the performance of the UMR-IW navigation system locks as the productivity of tows depends on the performance of system locks.

Definition 4: There are 38 *geographic regions* defined in the model. This number is small enough to make the model manageable and large enough to permit the analysis of potential actions to alter the performance of the 43 locks at the 37 system dams. The geographic regions are defined with respect to river origins and destinations relative to the UMR-IW system. There are 29 geographic regions defined by the lock and dam pools on the Mississippi River, eight geographic regions defined by the lock and dam pools on the Illinois Waterway, and one geographic region defined as the rest of the world. The navigable tributaries in the study area are included in the appropriate main stem pools. There are then 1444 unique water origin and destination pairs represented in the model. Normally, a spatial price equilibrium model requires detailed information regarding inland geographic regions and the transportation modes and links that service the inland geographic regions. However, in the model produced for this study, the inland geographic regions are mapped to UMR-IW waterside origins and destinations. This is done by aggregating producers' supply and demand functions in a consistent manner into the shippers' induced demand functions for water origin and destination specific transportation.

Definition 5: There are literally hundreds of different commodities currently moving on the UMR-IW navigation system. To reduce the size of the model, the commodities are aggregated into 11 *commodity groups*. The aggregation is based on product similarities with respect to end use and equipment utilized for water transportation. The 11 commodity groups are corn, soybeans, wheat, other farm products, coal, petroleum and related products, industrial chemicals, agricultural chemicals, iron and steel products, aggregates, and other miscellaneous commodities. The other farm products group is partitioned into down-bound and up-bound sub-groups. The agricultural chemicals group is partitioned into dry and liquid sub-groups. Note that the definitions of geographic areas and commodity groups together yield a total of 15,884 origin, destination, and commodity group combinations.

Table 1

Commodity Groups	1992 WCSC Tonnage
Corn	32,707,000
Coal	22,583,000
Aggregates	18,932,000
Petroleum and Related Products	13,968,000
Oilseeds	12,474,000
Iron and Steel Products	9,360,000
Miscellaneous Products	7,114,000
Wheat	5,870,000
Fertilizers	5,223,000
Industrial Chemicals	4,766,000
Other Grain Products	1,730,000
Total	134,727,000

Table 2

Study Regions
Upper St Anthony's Falls
Lower St. Anthony's Falls
Upper Mississippi River Pool 1
Upper Mississippi River Pool 2
Upper Mississippi River Pool 3
Upper Mississippi River Pool 4
Upper Mississippi River Pool 5
Upper Mississippi River Pool 5A
Upper Mississippi River Pool 6
Upper Mississippi River Pool 7
Upper Mississippi River Pool 8
Upper Mississippi River Pool 9
Upper Mississippi River Pool 10
Upper Mississippi River Pool 11
Upper Mississippi River Pool 12
Upper Mississippi River Pool 13
Upper Mississippi River Pool 14
Upper Mississippi River Pool 15
Upper Mississippi River Pool 16
Upper Mississippi River Pool 17
Upper Mississippi River Pool 18
Upper Mississippi River Pool 19
Upper Mississippi River Pool 20
Upper Mississippi River Pool 21
Upper Mississippi River Pool 22
Upper Mississippi River Pool 24
Upper Mississippi River Pool 25
Upper Mississippi River Pool 26
Upper Mississippi River Pool 27
Lagrange Pool
Peoria Pool
Starved Rock Pool
Marseilles Pool
Dresden Island Pool
Brandon Road Pool
Lockport Pool
Thomas O'Brien Pool
The Rest of the World

Postulate 1: The time frame of analysis for the model is one calendar year. The model identifies a UMR-IW navigation system equilibrium conditional on carrier supply and shipper demand functions for a given year. To estimate the system equilibrium conditions through time, new supply and demand functions representative of the year of analysis are introduced and a new equilibrium is estimated. Consequently, the model is not dynamic in the sense of relating system equilibria through time. This is a potential shortcoming of the model. Dynamic concerns must be addressed outside the system model.

Postulate 2: The performance of the 43 locks at the 37 lock and dam sites in the UMR-IW navigation system are characterized by the first two central moments of their service time distributions. The service time distributions of system locks are assumed independent of the service time distributions of other system locks.

Postulate 3: The expected total transit time at each system lock is equal to the expected process time at the lock plus the expected delay time at the lock. The upper bound of expected delay time at a system lock is approximated by the following mathematical relationship:

(1) $Ex(D) \;\#\; [(:_s^2+\Phi_s^2)/2(:_a-:_s)][(\Phi_a^2+ \Phi_s^2)/(\,:_a^2+ \Phi_s^2)]$, where $:_s$ represents the mean service time, Φ_s represents the standard deviation of the service time, $:_a$ represents the mean inter-arrival time at the lock, Φ_a represents the standard deviation of the inter-arrival times at the lock. An equivalent formulation of this upper bound and the conditions under which the bound is exact are presented in Marchal W. G., *Some Simple Bounds and Approximations in Queueing*, Technical Memorandum Serial T-294, Institute for Management Science and Engineering, The George Washington University, 1974.

Postulate 4: The distribution of tow sizes within individual commodity groups does not change over time. In other words, tow sizes are dictated by variables other than the performance of the locks. This postulate permits relating the demand for commodity flows to potential tow arrivals at system locks.

Postulate 5: Goods are homogeneous within commodity groups and between geographic regions. For example, soybeans are soybeans wherever they are produced or consumed.

Postulate 6: Carriers' supply functions and shippers' derived demand functions are well defined and continuously differentiable. Carriers' supply functions are non-decreasing functions of the price of water transportation. Shippers' derived demand functions are non-increasing functions of the price of water transportation.

Postulate 7: Carriers attempt to maximize profits. Furthermore, carriers do not collude when setting supply levels. Carriers attempt to maximize profits by, first, minimizing the costs of producing a given vector of outputs, and, second, determining the profit maximizing levels of outputs to provide. In attempting to maximize their profits, carriers allocate their scarce supply of tows in such a manner that the imputed marginal revenue products amongst possible uses of the tows are equal to the incremental costs of allocating the tows to those uses. This is an important result and useful in understanding the role of back-hauls in generating carrier profits. A back-haul is a commodity movement that occurs in equipment that otherwise would have moved empty anyway. Back-hauls are priced differently from front-hauls and, consequently, respond differently to potential system congestion.

Postulate 8: Carriers' decisions regarding operations and use of other inputs per unit of output are not affected by operating conditions at system locks. However, the number and allocation amongst competing uses of towboats and barges provided by carriers are affected by the performance of the system locks. Appendix B presents a detailed examination of the factors that can influence the allocation of tows by a carrier amongst competing demands.

Postulate 9: Shippers possess no market power in the market for inland waterway transportation. That is, shippers regard the price of inland navigation as given to them when making routing or quantity decisions. Furthermore, shippers do not collude when competing for the services of carriers.

Postulate 10: Shippers attempt to minimize the costs of shipping goods between every origin, destination pair for whatever quantity of goods they ship between origin, destination pairs. Shippers attempt to maximize their own profits by shipping goods between origins and destinations to the point that additional volumes produce no additional net revenue.

Postulate 11: The commodity markets in each region for each good are purely competitive. The shippers take the price of a good as given in a geographic region when deciding the quantity to export or import to a region.

Postulate 12: There are no structural changes in supply and demand functions in a period of analysis. That is, equilibrium prices and quantities are determined along fixed supply and demand functions. The supply functions of carriers are, however, related to the performance of the locks. This makes the model a partial spatial price equilibrium model.

Postulate 13: If consumption (here consumption refers to the use of an intermediate product by a producer) of a good occurs in a geographic region, then the market price of the good in that region is equal to the demand price of consumers in that region. If consumption of a good does not occur in a geographic region, then the demand price of consumers in that region is less than the market price of the good.

Postulate 14: If production of a good occurs in a geographic region, then the market price of the good in that region is equal to the supply price of producers in that region. If production of a good does not occur in a geographic region, then the supply price of producers in that region is greater than the market price of the good.

Postulate 15: If a good is transported from one geographic region to another geographic region on a specific transportation link, then the price of transportation on that transportation link is equal to the difference in market prices of the good between the importing region and the exporting region. If a good is not transported on a transportation link from a geographic region in which it is available to another geographic region, then the price of transportation on that link is greater than the difference in market prices between the regions. This postulate is the heart of spatial economic price theory. Its validity is crucial to the system NED model developed for this study. There is evidence to suggest that this postulate is a good representation of existing commodity flows occurring in the UMR-IW study area. The USDA maintains a database of weekly agricultural product prices for selected locations. This database also contains weekly price information on spot water transportation prices from inland ports to the port of New Orleans, LA. The data in these time series supports the validity of Postulate 15.

Figure 4, below, shows the weekly price of corn at St. Paul, MN, St. Louis, MO, and New Orleans, LA for the period 1990-1994. Most waterborne movements of corn on the UMR-IW navigation system originate in the study area and terminate in southern Louisiana ports destined ultimately for export. Note that the price of corn at New Orleans is greater than the price in St. Louis, which, in turn, is greater than the price in St. Paul. The relationship between these prices is stable regardless of the absolute levels. The water transportation cost between St. Paul and New Orleans is greater than the water transportation cost between St. Louis and New Orleans. As there are many movements of corn to New Orleans from both of these UMR-IW origin ports, this is exactly the price pattern spatial price theory and Postulate 15 would suggest should be observed. This is true for all other important agricultural locations and products in the study area. In fact, during the period 1990-1994, the correlation coefficients between weekly corn prices at the major UMR-IW system origins and prices at New Orleans are all greater than 0.95. The same is true for the correlation coefficients for weekly soybean prices at UMR-IW ports and soybean prices in New Orleans.

Figure 5, below, presents the relationship between the weekly waterborne delivered price of corn from St. Louis to New Orleans and the price of corn in New Orleans for the period 1990-1994. Note the remarkable correlation between the delivered price of corn and the price of corn at New Orleans. The same high correlation between delivered prices and the price in New Orleans is observed for all major agricultural commodities and origins in the UMR-IW study area.

Postulate 16: The sum of the quantity of a good consumed in a geographic region and the quantity exported from that region must be less than or equal to the quantity of the good produced in the region.

Postulate 17: The sum of the quantity of a good produced in a region and the quantity of the good imported by the region must be greater than or equal to the quantity of the good consumed in the region.

Figure 4

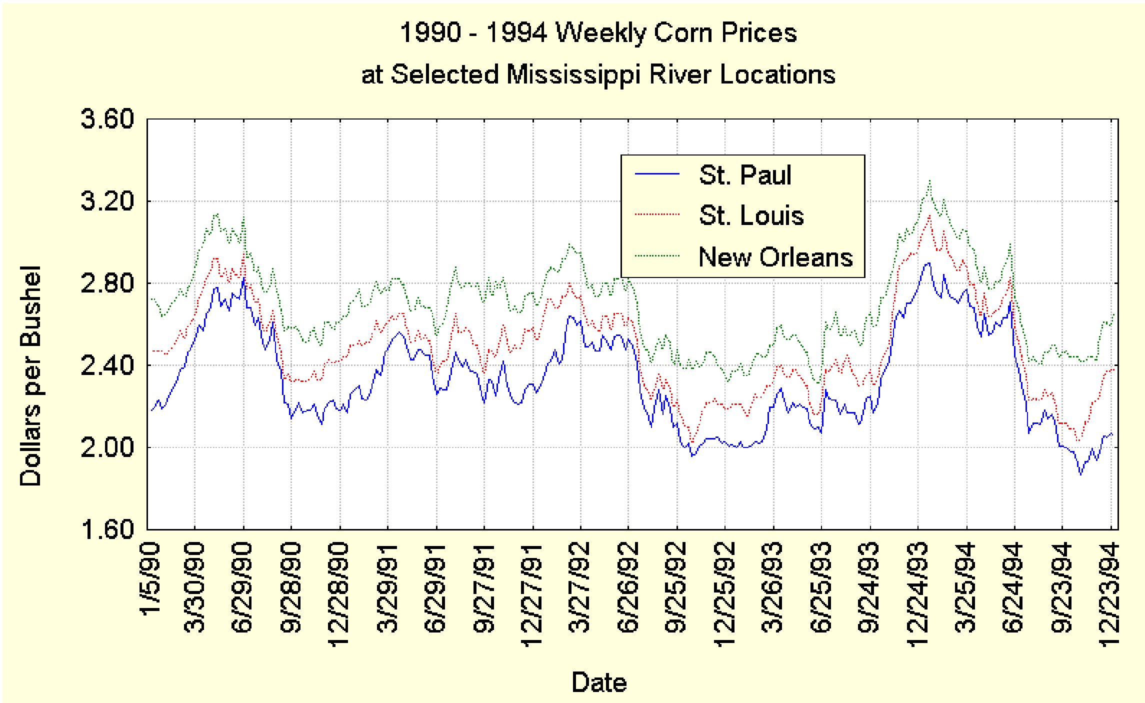
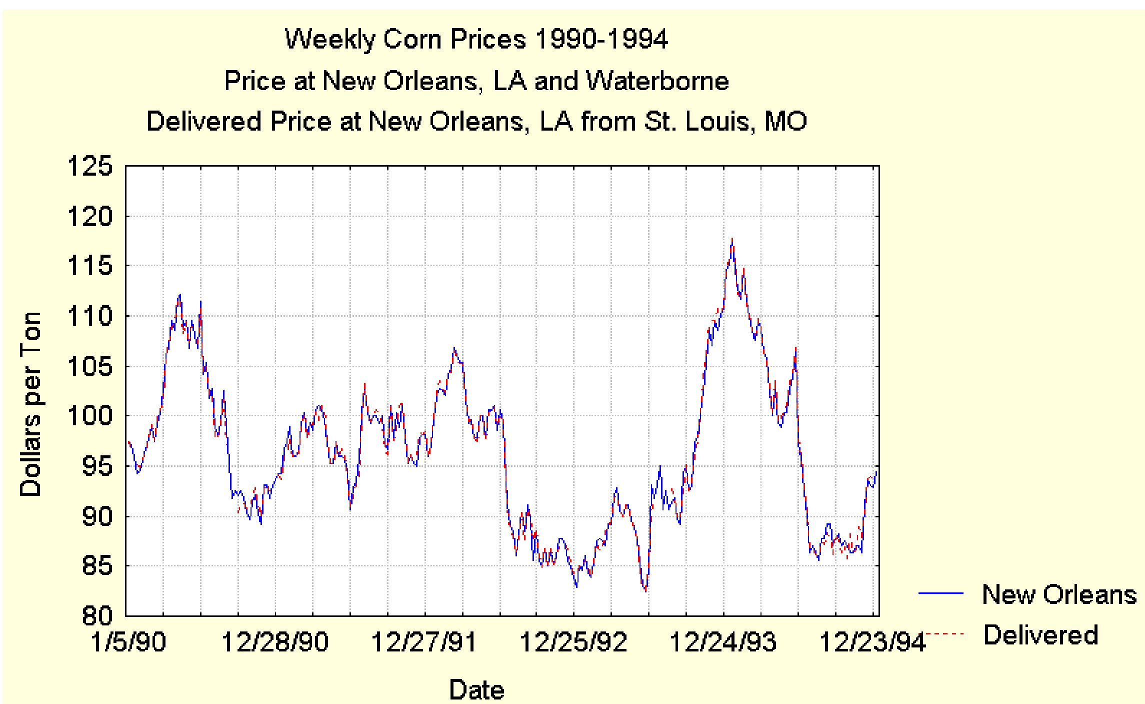


Figure 5

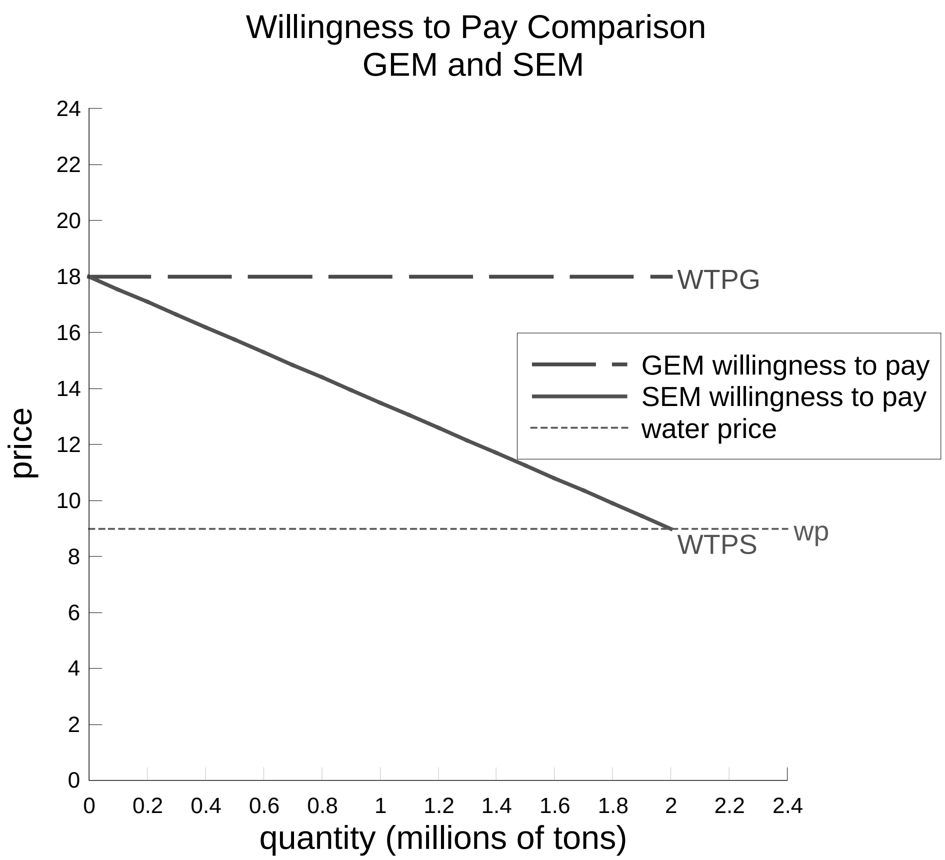


Utilizing these and other definitions and assumptions, a spatial price equilibrium model focusing on the derived demands of shippers and the supply functions of carriers is developed to measure the net beneficial impacts of potential changes to the UMR-IW navigation system. The model is similar in many respects to the General Equilibrium Model (GEM) used by the Corps of Engineers in previous navigation system studies. There are, however, two important conceptual differences between the GEM and the system model developed for this study.

The first conceptual difference is that the spatial price equilibrium model employed in the UMR-IW Navigation System Feasibility Study implements the economic postulate that *every* commodity, origin, and destination combination must have the quantity of waterway transportation demanded equal to the quantity of waterway transportation supplied to it at the equilibrium transportation price for that commodity, origin, destination combination. In other words, the willingness to pay for the last incremental unit of a system movement in equilibrium is equal to the market price of the private transportation resources required to accomplish that movement. The GEM model, and all other system models used by the Corps, did not require this condition for a navigation system to be in equilibrium. The GEM and other Corps navigation system models assumed that all potential tonnage from each specific origin, destination, commodity group combination had equal willingness to pay for water transportation. This willingness to pay was typically measured by the cost of the next cheapest mode of transportation between the same origin and destination. Spatial price equilibrium theory indicates that this is extremely unlikely to be the case as then the last incremental ton of each movement would have a willingness to pay much greater than the price it does pay for water transportation. This is a fundamental flaw in previous navigation system economic models. The cost of the next cheapest mode of transportation between the same origin and destination employed by the GEM and other Corps inland navigation economic models represents only an upper bound on the willingness to pay of a potential movement for water transportation. Further, this upper bound may have no relevance on the real willingness to pay for incremental units of water transportation other than limiting the willingness to pay to levels below that of the costs of alternative modes between the same origin and destination.

Figure 6 graphically depicts the difference between the representation of the willingness to pay in previous navigation system models, the curve labeled WTPG, and the representation of the willingness to pay in the SEM model, the curve labeled WTPS for a hypothetical system movement. This hypothetical movement of two million tons has a maximal willingness to pay of $18.00 (the cost of transportation on an alternative mode from the same origin to the same destination) and incurs an observed equilibrium water transportation price of $9.00. Note that the WTPS curve is drawn to intersect the existing water transportation price at the existing quantity. This is a requirement of the myopic equilibrium for this movement. If this were not the case, then the shipper could increase his own welfare by adjusting the quantity purchased at the existing water price. If the willingness to pay is greater than the price, the shipper will purchase more units and increase his welfare. If the willingness to pay is less than the price, the shipper can increase his welfare by purchasing fewer units. The WTPG representation of willingness to pay cannot represent a myopic equilibrium for this shipper unless his behavior is constrained external to this transportation market. In summary, at the margin, the WTPG curve is unlikely to represent the willingness to pay for incremental units of output for this movement.

Figure 6



Spatial price equilibrium theory further suggests that the maximum contribution to NED of each potential origin, destination, commodity group movement is the *minimum* willingness to pay to avoid *all* other alternatives to waterway transportation. For alternatives involving a different mode of transport to the same origin and destination, the maximum willingness to pay is the transportation costs on the alternative mode. For alternatives involving water transport to a different destination, the maximum willingness to pay is the loss of net income to the producer of the commodity. For alternatives involving water transportation from a different origin, the maximum willingness to pay is the loss in net income to the ultimate consumer. For alternatives involving no movement of the commodity, the maximum willingness to pay is the net loss in income in the producing region. The minimum of all alternatives to the existing water transportation routing represents the maximum willingness to pay for water transportation of a potential system commodity movement.

The navigation system model created for the UMR-IW distributes the willingness to pay for water transportation between the maximum willingness to pay for the first ton of a movement and the minimum willingness to pay of the last incremental ton of potential system movements. The study dedicated significant resources to identify the maximum and minimum willingness to pay for UMR-IW navigation system use. This work was contracted to and completed by the Tennessee Valley Authority (TVA). A stratified sample consisting of 1331 unique UMR-IW movements was randomly selected from the 1992 Waterborne Commerce Statistical Center detailed barge data file. This sample was provided to the TVA to identify the costs for water transport on the existing water routing, the alternatives to water transportation for each movement, and the costs of those alternatives. The TVA data, expressed in 1994 price levels, were then extrapolated to the full population of over 79,000 individual barge movements. Details of the method used to extrapolate the TVA data are presented in Appendix C. The 79,000 individual movements and the TVA cost data were subsequently aggregated into the 38 geographic regions and 11 commodity groupings to yield information used in the model on the maximum and minimum willingness to pay for water transportation between the regions for each commodity group. Note that, even with this level of aggregation, there are 15,884 (38x38x11) unique maximum and minimum willingness to pay combinations defined in the model.

In contrast with the significant quantity of study resources directed at identifying the maximum and minimum willingness to pay for each system movement, the study directed no resources to identify the distribution of willingness to pay between these two values. This was an important oversight in the original scope of the economic efforts for this study. ER 1105-2-100 suggests that in the absence of more detailed information that the willingness to pay be equally distributed between the maximal and minimal values. Consequently, for all existing movements of all commodity groups, with the exception of agricultural products, the willingness to pay for waterborne transportation is linearly distributed between the maximum and the minimum values. For agricultural products, the state of Iowa provided to the study team a report entitled “The Iowa Grain Flow Survey: Where and How Iowa Grain Producers and Country Elevators Ship Corn and Soybeans” published by Iowa State University, 1996. This report presents detailed information regarding 1994 flows of corn and soybeans from producing regions in Iowa (the nine Iowa Crop Reporting Districts) to Mississippi River destinations as well as other destinations for these products. This data in conjunction with data provided to the study team by the United States Department of Agriculture on county level prices received by agricultural producers in Iowa from 1989 through 1994 supports a quadratic form to represent the willingness of users to pay for water shipment of these products. The USDA data indicate a real world price differential to grain producers of approximately $5.00 per ton moving east to west away from the Mississippi River through the state of Iowa for the period 1989 through 1994. The Iowa flow data indicates that the majority of grain destined to Mississippi River ports originating in the state of Iowa comes from very near the river itself. Figure 7 presents the percentage of corn delivered to Mississippi River ports as a function of distance to the river. Figure 8 presents the same information for soybeans.

Figure 7

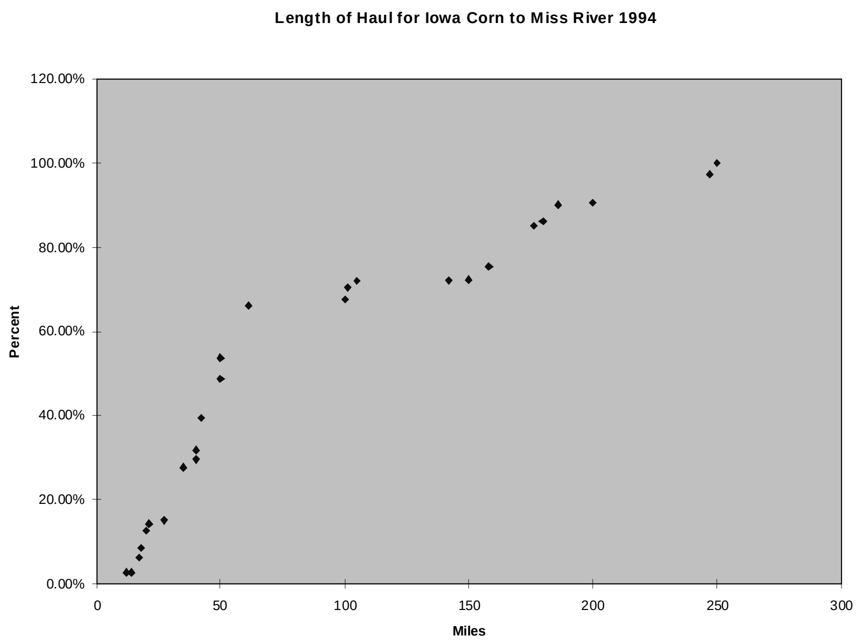
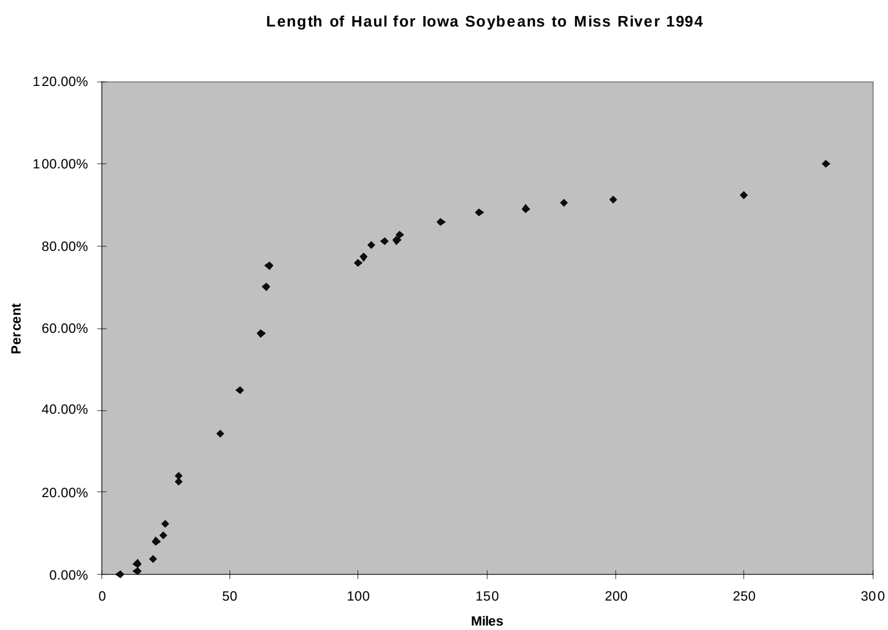


Figure 8



Tables 3 and 4 below summarize some of the data contained in the Iowa Grain Flow Survey for 1994 corn production. Table 3 shows the percentage of total corn production for each Iowa Crop Reporting District destined for Mississippi River terminals. Table 4 displays the same information for corn destined for processing.

Table 3	*Iowa Crop Reporting Districts*	*Percentage of 1994 Corn Production*	*Destined for the Mississippi River*
	West	Central	East
North	11.7	32.2	58.2
Central	4.0	9.3	47.0
South	4.5	2.7	47.1

Table 4	*Iowa Crop Reporting Districts*	*Percentage of 1994 Corn Production*	*Destined for Corn Processors*
	West	Central	East
North	23.7	48.4	36.1
Central	56.9	63.2	46.3
South	16.6	74.2	40.6

Note that in Table 3 the percentage of corn produced in the crop reporting districts destined for Mississippi River terminals decreases at a rate increasing with distance from the river. This phenomenon is consistent with a spatial price equilibrium outcome with distance to a Mississippi River terminal serving as a proxy for the willingness to pay to access inland waterborne transportation. This data supports a less than linear relationship between the willingness to pay for waterborne transportation and the quantity supplied to waterborne transportation.

Table 4 indicates that there is a significant non-waterborne transportation alternative currently selected by agricultural producers throughout the state of Iowa. Substantial quantities of corn produced in all the Crop Reporting Districts are destined for corn processing plants. Hence, there is a shift in destination alternative available for producers currently using the Mississippi River. Further, the Iowa Grain Flow Survey indicates that there are negligible quantities of corn production in the state of Iowa that currently move to the same ultimate destinations as those served by the Mississippi River. This suggests that for corn currently moving to the Mississippi River, utilizing a different mode of transportation to the same ultimate destination is not a real alternative to waterborne transportation. It appears far more likely that a change in destination is the real alternative for corn destined to Mississippi River terminals.

The Iowa Grain Flow Survey also contains detailed information regarding the production and flow of soybeans in the state of Iowa. The data for 1994 soybean flows mirrors that presented above for corn flows. Tables 5 and 6 below present the soybean flows to Mississippi River terminals and soybean processors, respectively.

Table 5	*Iowa Crop Reporting Districts*	*Percentage of 1994 Soybean Production*	*Destined for the Mississippi River*
	West	Central	East
North	8.6	8.5	50.9
Central	0.8	2.3	55.4
South	5.2	21.5	77.0

Table 6	*Iowa Crop Reporting Districts*	*Percentage of 1994 Soybean Production*	*Destined for Soybean Processors*
	West	Central	East
North	79.2	82.8	39.5
Central	89.2	78.8	40.0
South	44.5	75.9	16.5

The second important conceptual difference between the system NED model developed for this study and all other Corps inland navigation system economic models is evidenced in the estimation of the relationship between realized system traffic levels and the quantities of privately supplied carrier resources required to produce these outputs. The model developed for this study explicitly accounts for the fact that there are a finite number of available tows at any point in time. This fact has two important economic consequences.

First, the queue size at any individual system lock at any point in time cannot increase without bound. There are only a finite number of tows in existence at any point in time, hence lock queues cannot increase beyond levels determined by the operation of the tows. Consequently, significant increases in system levels of delay require a change in tow operations or an increase in the number of tows operating in the system. For either of these two variables to change, system shippers must be willing to pay for the change in operations or the additional equipment, and system carriers must be willing to purchase the equipment or change their operations.

There are two cases of inland navigation system locks that evidenced significant congestion for periods of ten years or more. Old Lock and Dam No. 26 near St. Louis, MO, in the Upper Mississippi River and the Inner Harbor Navigation Canal Lock, New Orleans, LA, in the Gulf Inter-coastal Waterway. Lock and Dam No. 26 was recently replaced by the Melvin Price Locks and Dam and the Inner Harbor Navigation Canal Lock is still operational and congested. Figure 9 displays a time series of the number of tows processed and the annual mean processing delay at Lock and Dam No. 26 for the period immediately preceding its replacement. Figure 10 displays a similar time series of the number of tows processed and the annual mean processing delay at the Inner Harbor Navigation canal Lock through 1997. Note, that in both these time series once the lock in question congests to the level of 10 to 15 hours of average delay that the level of congestion does not seem to increase further. This is a very interesting result and yields further evidence on the willingness of shippers to pay and carriers to purchase equipment in the face of congestion.

Figure 9

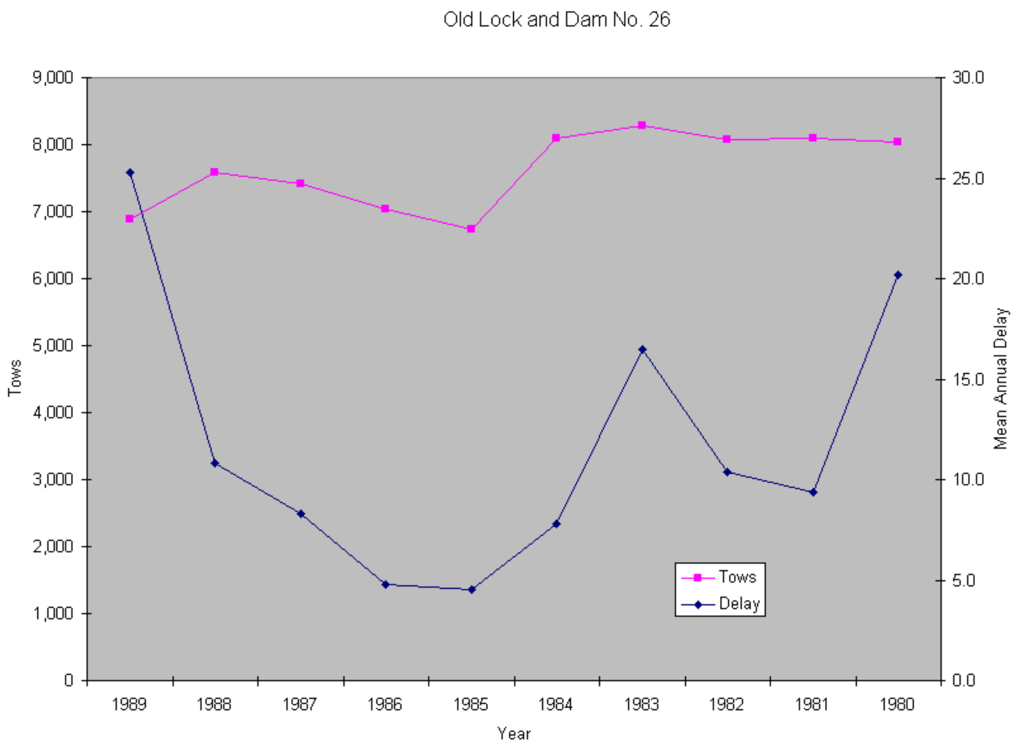
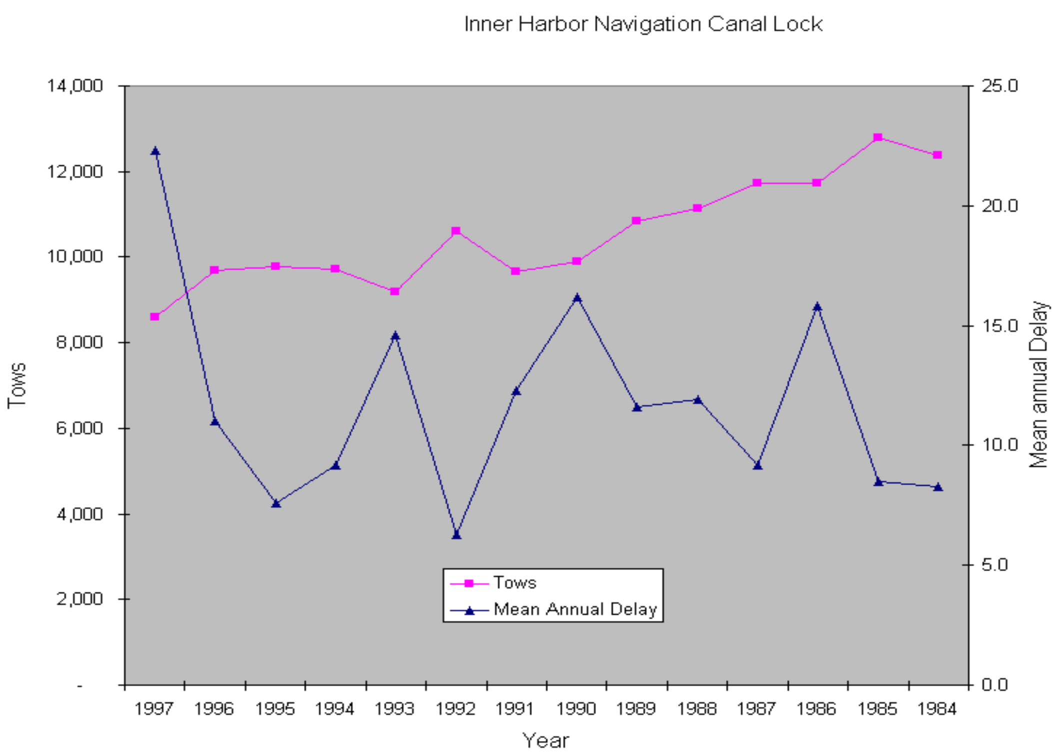


Figure 10



Existing navigation system models would forecast rapidly increasing levels of congestion and delay as users with no alternative to waiting would be forced to accept phenomenal increases in delay levels. This simply does not happen. Shippers appear to be unwilling to pay past the costs of these sustained levels of delay. Carriers will not invest in equipment unless shippers are willing to pay for it. Hence, real world congestion seems self-limiting.

Second, the inter-arrival times between tows at system locks depend on the service distributions of the other locks in the navigation system. The expected delay function described in Postulate 3 above describes the relationship between the upper bound of expected delay at a system lock, the distribution of inter-arrival times at that lock, and the distribution of service times at the lock. The distribution of inter-arrival times at a lock is related to the distribution of service times at other system locks used by arriving tows. Hence, the upper bound of expected waiting time at a lock is related to the performance characteristics of other system locks.

The NED navigation system model is implemented in a Microsoft Excel 5.0/Office95 spreadsheet and makes extensive use of the solver tool to solve systems of equations for an equilibrium state. The system equilibrium is characterized by a set of transportation prices and traffic flows for the origin, destination, and commodity group combinations. The systems of equations relate lock performance characteristics to tow productivity; tow productivity to transportation prices and carrier supply functions; transportation prices to the shipper derived demand functions; and forecasts of changes in unconstrained demand to shipper derived demand functions.

Appendix A presents detailed information regarding the data and computations performed in the Excel workbook itself. Appendix A also identifies additional simplifying assumptions used to create the spreadsheet model. The steps outlined below summarize the iterative execution of the model.

1. The model parameters are input replicating an observed system condition of flows, transportation costs, and lock performance. This observed condition is assumed to be an equilibrium observation for that year.

2. The NED benefits of the system are measured for the base year by computing the area between the estimated derived demand functions for transportation of shippers and the estimated supply of transportation functions of carriers.

3. The model results are then compared to the observed system behavior to verify their reasonableness.

4. Then the quantities demanded by shippers at existing transportation costs are increased to the levels forecast for the next period of analysis. The periods of analysis are 2000, 2005, 2010, 2020, 2030, 2040, and 2050. Lock performance parameters are adjusted to reflect the forecast of lock operations at that time.

5. A new forecast equilibrium is estimated for the changed demands and lock operating parameters. The NED benefits of the system are measured in the period of analysis by integrating under shipper demand curves and above carrier supply curves.

6. Then potential system actions are modeled by altering the performance or reliability characteristics of system locks reflecting the results of the actions. A new resulting system equilibrium is estimated reflecting carrier and shipper responses to the altered operating conditions. The NED benefits in the period are then re-estimated yielding the change in NED benefits resulting from the modeled system actions.

7. Steps 4, 5, 6, and 7 are repeated until all selected years of analysis are completed.

4. Model Data Requirements and Sources

The navigation system NED model requires data regarding the existing and forecast performance of system components, existing and forecast commodity flows, existing costs of system transportation, and alternatives to system transportation for existing shippers. The sources and vintages of data used in the NED model are described below. The details on use of the data in the spreadsheet are contained in Appendix A below.

Data Set	Vintage	Source
Existing Lock Performance	1992	LPMS
Forecast of Lock Performance	1997	Engineering Work Group
Existing Commodity Flows	1992	WCSC
Existing Transportation Costs	1994	Tennessee Valley Authority
Alternatives to Water Transport	1994	Tennessee Valley Authority
Forecast Commodity Flows	1996	Jack Faucett Associates
Future Relative Modal Costs	1998	Tennessee Valley Authority

5. Risk and Uncertainty Considerations

On August 15, 1994, the Director of Civil Works provided the study guidance for conducting navigation system analysis. In part, the guidance stated, “Since the reliability of system components, the environmental benefits and impacts, and the economic costs and benefits are not known with certainty a probabilistic risk-based analytical framework is essential for good decision making. Input to the analytical framework is important; all collected data are to maintained to preserve the characteristics of the distributions of the decision parameters so that the full range of possible effects on the economic, physical and environmental performance can be better understood, evaluated, and displayed.”

The NED navigation model developed for this study has only limited potential for accomplishing risk-based analysis. Steady state equilibrium models are ill-suited for measuring dynamic phenomena. Potential distributions of system equilibria can be explored by the probability weighting of important input parameters and examining the resulting distributions of equilibria. Dynamic phenomena such as extended periods of service disruptions, temporary random lock closures, and other similar events cannot be directly examined using the system model.

6. Other NED Effects

ER 1105-2-100 provides for the measurement of other direct NED benefits which are incidental direct effects of a project or plan that increase economic efficiency and are not otherwise accounted for in the evaluation of the pan or project. They are incidental to the purposes for which the water resource plan is being formulated. They include incidental increases in the output of goods and services and incidental reductions in production costs. For example, the implementation of a measure designed to reduce congestion at a UMR-IW navigation system lock could have the incidental effect of decreasing the amount of resources required to treat fuel emissions of alternative modes of transportation by inducing shippers to use the navigation system more intensively than they otherwise would have. Since there is no market mechanism to allocate emission treatment costs to individual economic agents creating these costs transportation market prices do not capture these NED effects.

Another NED effect described in ER 1105-2-100 is the use of otherwise unemployed or under-employed labor resources. The notion here is that these type of labor resources have smaller opportunity costs associated with their usage than the wages they would receive for implementing the plan.

Finally, expenditures for operation, maintenance, and rehabilitation of the existing system can be changed by potential system actions. For example, replacing an older chamber with a new chamber can avoid or reduce future rehabilitation related expenditures on the older chamber. These possibly reduced expenditures are potential NED consequences of system actions.

The navigation system NED model created for the UMR-IW navigation system feasibility study does not capture either of these potential NED effects. These other NED effects are measured by other economic models created for this study.

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